SEISMIC FRACTURE ANALYSIS IN CONCRETE GRAVITY DAMS

Deeja Alora, Indrani Gogoi

Abstract— Modelling of crack propagation in solids has been a major area of focus both in industry and research communities. The formulation and numerical implementation of an embedded finite element technique which incorporates the cohesive frictional law is presented here. This technique is also termed as embedded cohesive element method. The solution of dynamical system is obtained using the classical Newmark's Method. The crack is restricted to propagate from edge of one element to the other, only if the crack propagation criterion is fulfilled. A set of MATLAB codes, called MAT-DAM for convenience, have been developed particularly for the purpose of crack propagation in concrete gravity dams under static and dynamic loadings. A comparative study on dynamic analysis result is carried out between the dam with and without fracture. The problem definition arises through a cohesive frictional crack model using an EFEM approach.

Index Terms— Cohesive fracture, Crack model, Crack initiation, Crack propagation, MAT – DAM, Lift Joints, Embedded finite element

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1 INTRODUCTION

1.1 FRACTURE MECHANICS CONCEPT

A crack which is present in a loaded body can be deformed in different ways. Irwin observed that there are three independent kinematical movements of the upper and lower crack surfaces with respect to each other and these are categorized as: Opening mode, Shearing mode, and Tearing mode.

Crack model

Fracture is an important mode of deformation and damage in both plain and unreinforced concrete structures. To predict the accuracy of fracture behaviour, using finite element analysis would be essential. In this paper, for the process of crack propagation analysis in concrete structures, there are two general models: discrete crack and smeared crack [Wang et al. 2000].

Smeared crack model: Smeared crack method is based on two essential steps. The first step is to detect the place of initial crack, and the second one is to estimate the crack path, and to replace it with a soften element. The smeared crack approach implies a continuum type representation with a fixed Finite Element mesh. In this method, crack depends on the concrete materials and it will happen when the stress exceeds of allowable amount. The smeared crack model can consist of two parts: one is initial part of the crack that determines the orientation and location of a new crack, and the other one is the developed part where tractions and displacements of the crack opening is determined by the softening law [Lohrasbi et al. 2008].

Discrete crack method: Discrete crack method is known as natural crack model. Methods pertaining to the discrete crack approach calculate each crack individually in an explicit way in the Finite Element mesh. After pioneering works in which cracks would be allowed to open between exist continuum elements according to a maximum stress criterion, procedures for general crack propagation with remeshing were developed for concrete structures. Newer software techniques now enable the remeshing process, at least in two dimensional problems. The fracture process zone may be defined as the area surrounding a crack tip in which inelastic material behaviour occurs. In very large concrete structures (ex. dams) it is possible to apply linear elastic fracture method appropriately [Ahmadi et al. 2001].

A numerical scheme based on nonlinear crack band theory is presented to study the 2D seismic fracture behaviour of concrete gravity dams. A mesh size of the Finite elements close to the characteristic size of the crack band of concrete material is adopted, so that the strain softening behaviour of the concrete can be properly taken into account. Also, a technique of Finite element (FE) remesh at the crack front is presented by changing the element edge pairs of the cracking element candidate to be parallel with the principal tensile stresses, in order to better accommodate the crack extension. The procedure is verified using test results for a notched beam and then applied to the seismic fracture analysis of the Koyna dam in India as a demonstration of prototype application.

1.2 FINITE ELEMENTAL FORMULATIONS

The Strong Discontinuity Approach

In this section an introduction to the discrete model of the crack adopted in the present work is given. While embedding the cohesive crack model into finite elements two possibilities arise depending on the way cracks are modelled. If model of the crack is a band, then the thickness of the crack/discontinuity inside the element has to be considered, which give rise to the weak discontinuity. If the crack inside the element is of zero-thickness which is the model of the crack chosen for the present study, then the approach is called strong discontinuity approach. It's named so because of the oscillating

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character of the displacement field at both sides of the crack inside one element (technically).

Prior to the development of strong discontinuity approach, smeared crack models (band models) were used for numerical simulation of cracking in concrete. Despite the many theoretical flaws in these models, they are simple and with careful use they can provide remarkably good predictions of actual responses [Wells & Sluys, 2001]. The key issue is that the finite element size must be included directly in the constitutive model to make the energy dissipated in failure objective. This numerical shortcoming is termed the "spurious mesh transfer", which is the finer the mesh size the less the energy dissipated [Roy Chowdary & Narasimhan, 2000].

On the other hand discrete or cohesive crack approaches indeed has led to the development of elements with embedded strong discontinuity. Unlike the smeared crack models which use the element size as the length scale for hardening modulus [Hughes, 1987]; the length scale does not appear in finite elements with strong discontinuity and they are thought as zero-thickness surface. This is the reason why these approaches are called "strong discontinuity" (i.e. a sudden displacement jump which is entirely localized in a surface). In fact it is shown in that there is a conceptual equivalence between smeared and discrete crack approaches. This will be discussed later when we present the discretized form of embedded finite elements with strong discontinuity. Alternatively, the use of special interface elements to capture this displacement jumps has also been reported in the literature, but as mentioned in the previous section they require a priori knowledge of crack path.

In the case of the derivation of element matrices obtained from global coordinates, involves the integration of shape functions and their derivatives or both over the element. The integrals can be evaluated easily if the displacement models are written in terms of natural coordinate system or local coordinate system that is defined separately for each element [Bathe. 1996, Hughes, 1987].

Once [k], [m] and [c] matrices are calculated for each element locally, the global stiffness matrix [K], global mass matrix [M] and global damping matrix [C] are calculated for the whole system. The calculation of these global (system) matrices depends on the element connectivity. The algorithm for their calculation is given in the MAT-DAM pre-processor code.

Dynamic Analysis By Newmark's Implicit Integration Method

In the case of static analysis, where inertial and damping effects are not active, the solution of the boundary value problem ultimately reduces to the following linear system of equations:

$$[K]\{u\} = \{f_{ext}\}$$

where $[K]$ is the global stiffness matrix after assembling the stiffness matrices of all the elements. In dynamic analysis where both inertial and damping effects are present the equation of motion has to be linearized. Using the classical Newmarks method first the equation of motion is linearized and is solved the same way as in the case of static analysis. Once the linear system of equations is solved (in static or dynamic analysis), the stresses can be calculated. In the context finite element analysis the stiffness, mass, damping and stresses values are computed by numerical integration technique rather than the exact analytical solution. In this work the Gauss Quadrature technique is adapted for calculation of stresses.

The Cohesive Fracture Law

The cohesive fracture constitutive law which is used separately for the crack is essentially the crack initiation criterion, because in the present FE formulation no information is provided for the crack initiation from the bulk. The Mohr-Coulomb friction model generally used for geo-materials and concrete can now be written for the Eqns. 4.15 and 4.16, which implies that discontinuity can carry a certain magnitude of shear stress across their interface before they start "sliding" relative to each other. This state is also known as "sticking".

$$\tau lim = c + \mu \sigma max$$
$$|\tau max| < \tau lim$$

The τmax values calculated at each node will be compared with the limiting value of shear stress for each time step and if the maximum shear stress at a node exceeds, then the node is considered failed [Carol et al. 1997 and Cai et al. 2008].

Crack Propagation and Crack Path Continuity

Although stresses are most accurate at gauss points, but due to computational and round-off errors, most of the time stress values in gauss point oscillate, and the failed points may not lie next to each other. The problem becomes even more complicated in dynamic analysis. The neighbouring nodes and consequently the neighbouring elements may not fail at subsequent time steps [Alfaiate et al., 2002]. A common technique to deal with the crack propagation is to first identify the damage zone i.e. the elements which have their four nodes/three nodes/two nodes failed and mark these elements as failed elements, then order the these elements in such a way that a proper crack path continuity can be enforced. The crack path continuity algorithm used for the present work is explained in the following:

1. Based on the crack criterion (Cohesive fracture law) first the damage zone is identified, i.e. the cracked elements at each time step.
2. In the damage zone the location of the interpolation point is found out with the maximum shear stress value to be the crack initiation point, CI.
3. Now knowing CI we need to get the other points of discontinuity/crack in the damage zone. In the proximity of CI we identify edges of elements which have the highest shear stress values.
4. The next point will be located on the identified edge of the elements. The location will be based in the weighted average of the shear stress values of the nodes on the element edge. This point will be K1.
5. We repeat steps 3 and 4 again till the crack propagates to an element with only two failed nodes. This will give points K2, K3, etc.

6. The whole process is repeated at each time step with the last crack point from the previous time step becoming the initiation point for the present time step.

Flow chart for the analysis and routine used in MAT-DAM is shown below.

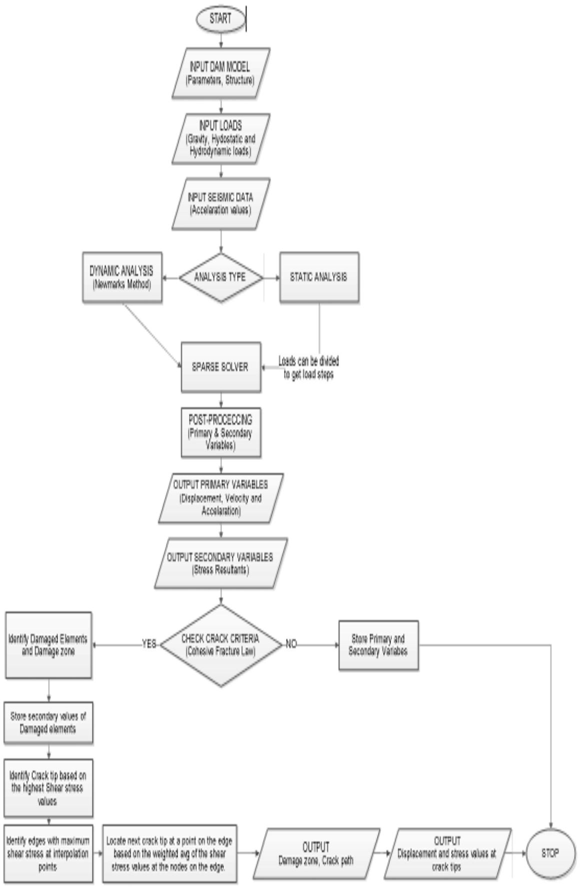

Fig. 1. Flow chart for programing

The work of Bazant reveals that if the size of the dam exceeds a certain limit, the apparently conservative ‘no tension’ design cannot always be regarded as safe. The rigid body equilibrium, strength based criterion was initially adopted where it was assumed that a crack would propagate whenever the principal tensile stress at the crack tip exceeds the specified tensile strength of the concrete. This was the only criterion for determining crack growth in concrete dams before the late 1970s. The strength-based criterion for crack analysis of concrete dams is based on the assumptions that there is a linear distribution of compressive stresses in the un-cracked concrete, and that a crack will propagate horizontally in a plane and extend up to a point where the tensile stress becomes zero.

2 CRACK PROPAGATION IN PINE FLAT DAM UNDER DYNAMIC EXCITATION

The material parameters used for the analysis are reported in [Mao & Taylor 1997], E = 30 GPa, υ = 0.2, cohesion value is assumed zero for capturing the crack path [Alfaiate et al. 2005] and the friction coefficient is taken as 0.577. The damping value considered is 5%. Boundary condition used is fixity at bottom.

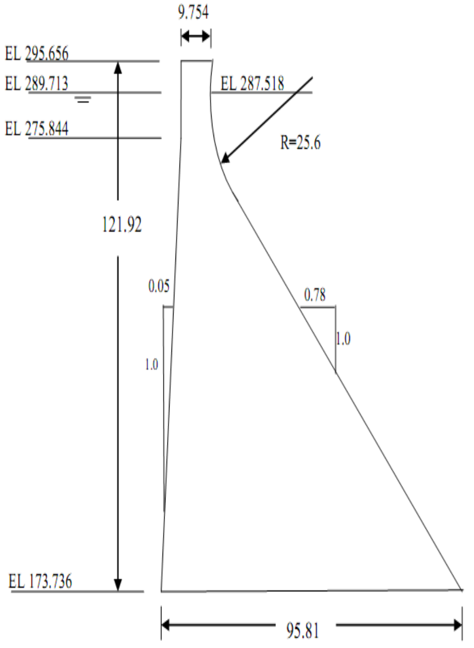

Fig. 2 Pine Flat Dam Structure

For the seismic analysis of Pine Flat dam, the horizontal component of San Francisco 1989 earthquake acceleration record downloaded from PEER Berkeley website has been applied to the base of the dam. The San Francisco 1989 earthquake was a devastating earthquake of magnitude 6.9 which hit the Loma Prieta on October 17, 1989, at 5:04 p.m. local time. The maximum gravitational acceleration of the ground recorded was around 3.5g. The FE model shown above is the one from ANSYS in which the base is restrained and the acceleration is applied to the entire model [ANSYS 2002]. It has to be noted that, the results (displacements, velocities, accelerations, and stresses) of the MAT-DAM code for the present E-FE analysis have been simultaneously cross-checked with ANSYS to ensure that the implementation is correct.

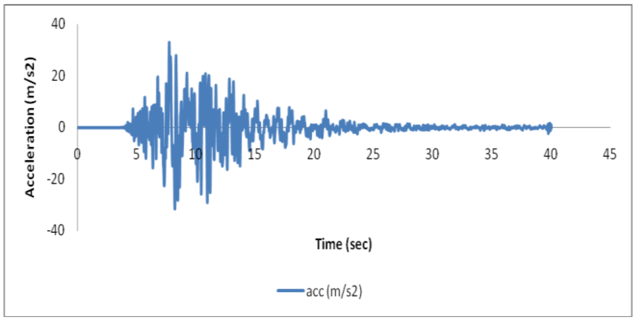

Fig. 3 Accelerogram for San Francisco 1989

The dam was analyzed with gravity loads, hydrostatic pressure = ρgh and hydrodynamic pressure calculated from IS 1893 applied at upstream. At the 7th time step the crack initiates at the downstream near the neck. The pattern of the crack obtained is mixed mode. MAT-DAM code result is now combined for the second case. The crack path after dynamic analysis is shown in Figure.

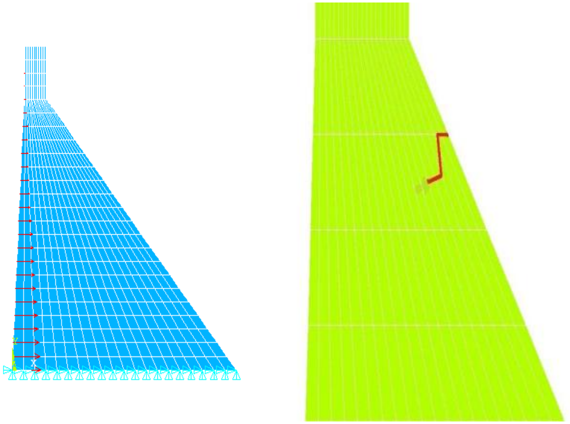

Fig. 4 Pine Flat FE Model & crack path

TABLE 1
COORDINATES, DISPLACEMENT AND SHEAR STRESS AT THE CRACK POINTS

Points	X co-ord (m)	Y co-ord (m)	Disp X (m)	Disp Y (m)	Shear Stress (N/mm^2)
1	18.5675	97.003	0.0089	0.0013	2.7917
2	17.8758	97.003	0.0089	0.0014	2.5365
3	19.1839	94.6274	0.0089	0.0013	2.2152
4	18.4571	94.5128	0.0086	0.0013	1.9623
5	17.6928	94.4432	0.0086	0.0014	1.9914

Now the first two crack points are selected and the crack is modelled with crack width of 0.003m in Pine Flat dam and dynamic analysis is carried out with the same loading, but for the crack model different meshing and element is used. Comparison of dam model with and without crack is given in following graphs (Figure 5A, 5B & 5C).

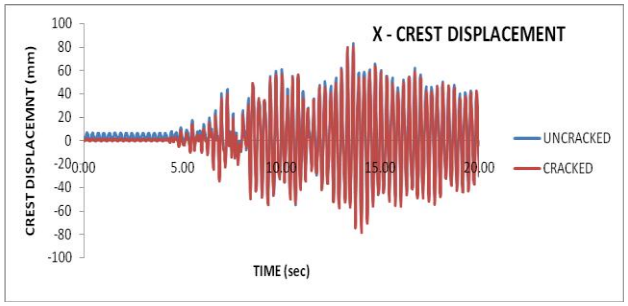

Fig. 5 (A) Pine Flat Dam – X crest displacement

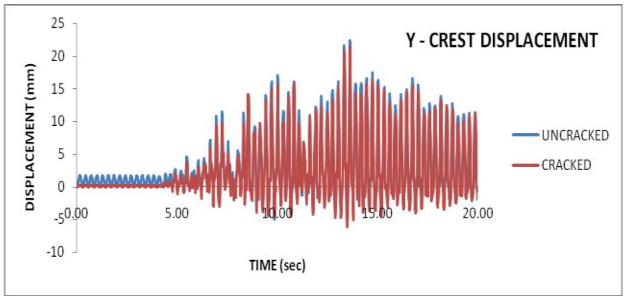

Fig. 5 (B) Pine Flat Dam – Y crest displacement

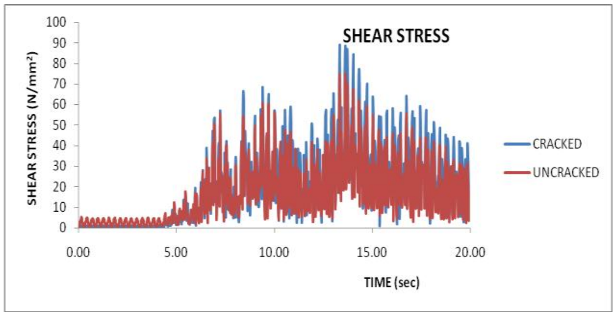

Fig. 5 (C) Pine Flat Dam – Shear Stress

The damage zone in the present analysis is relatively smaller than that reported by Mao & Taylor [1997]. This is due to the difference in material models and fracture laws used. The point of crack initiation at the neck is in the proximity of the crack initiation reported by Mao & Taylor [1997].

The advantages of present work over are:

1. The material model used in the present analysis is not mesh sensitive, whereas in [M. Mao & Taylor 1997] the fracture energy criterion is used to remove the mesh sensitivity of the analysis.

2. The material model used in the present analysis is a linear orthotropic model, with a separate fracture law for the crack. In [Mao & Taylor 1997] the nonlinear concrete model is used which as mentioned before is a smeared crack model for the concrete. Considering that the bulk is behaving inelastic in [Mao & Taylor 1997] and it is elastic in the present case, the computational cost can be significantly reduced.

3. The concrete model in ADINA is a continuum model, and the cracking pattern reported in [Mao & Taylor 1997] is based on failed elements whereas in the present analysis a discrete cohesive law is used for the crack which not only identifies the failed elements but also identifies the crack path.

3 CRACK PROPAGATION IN KOYNA DAM UNDER DYNAMIC EXCITATION

The geometry of a typical non-overflow monolith of the Koyna dam-reservoir-foundation is illustrated in Figure. This monolith is 103 m high and 70 m wide at its base [Chopra and Chakrabarti, 1973]. The upstream wall of the monolith is assumed to be straight and vertical which is slightly different from the real configuration. The depth of the reservoir at the time of the earthquake was 91.75 m. The non-overflow monolith of the dam is assumed to be in the plane-strain condition. Boundary condition used here is fixity at bottom. The effect of reservoir is applied as hydraulic pressure distribution on upstream side.

Parameters of the Koyna dam considered are [Bhattacharjee and Leger, 1993; Ghrib and Tinawi, 1995; Skrikerud and Bachmann, 1986]: elastic modulus E = 31027 MPa, mass density $\rho =$

2643 kg/m^3, Poisson's ratio $\nu = 0.2$. Dam model is subjected to Koyna 1967 horizontal acceleration which is applied to the base of the dam.

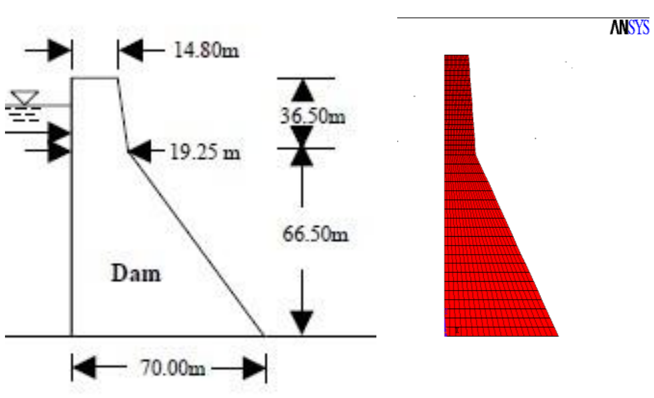

Fig. 6 Koyna Dam structure & FE model

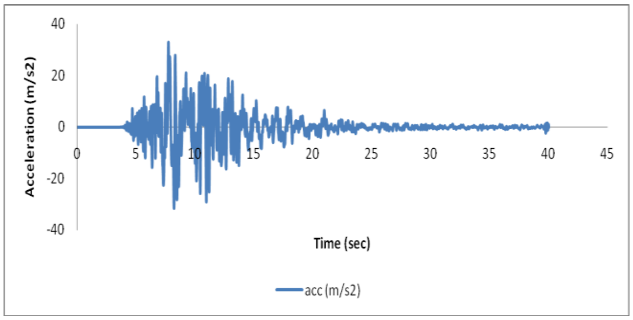

Fig. 7 Koyna 1967 Horizontal acceleration

The combined effect of water pressure and seismic acceleration is taken and transient dynamic analysis is done. Analysing the results the damage zone is identified. The 400th element has got damaged first. The damage zone was identified at 0.5 sec. and 4.2 sec. The associated nodes are 22, 43, 441 and 41. The first two nodes are selected and a horizontal crack of width 5mm was modelled. In the next stage Koyna with crack is modeled and analyzed. The results are compared and shown in following section. The results are quite match with [Sarkar & Paul 2007]. Table 5.7 shows a comparison of results with literature review. Slight difference is because the base excitation given in ANSYS is only in X direction and hydro dynamic effect is considered here where as in [Sarkar & Paul 2007] hydro static force and excitation in Y direction also were applied.

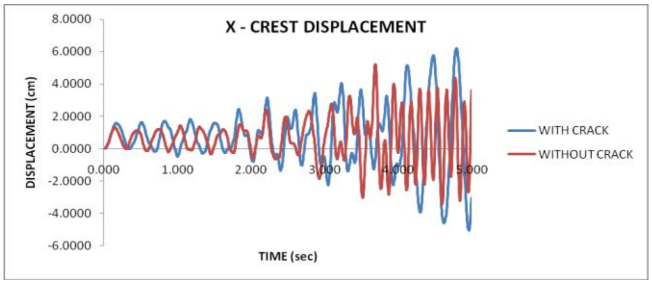

Fig. 8 (A) Koyna Dam – X crest displacement

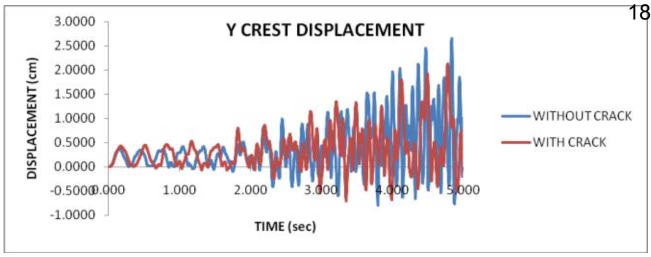

Fig. 8 (B) Koyna Dam – Y crest displacement

TABLE 2
KOYNA – RESULT VIEW

Cases	Crest displacement (cm).		Crest acceleration (m/s^2)		Heel stress (Mpa)
	Horizontal	Vertical	Horizontal	Vertical	
Sarkar, and Paul 2007	3.38	1.25	15.89	10.93	0.71
ANSYS	3.60	0.997	93	90	10.81

The nonlinear seismic analysis of a cracked concrete gravity dam has been investigated by different approaches to study the propagation of cracks in this kind of dams. The discrete crack approach, models the crack by separating the nodes of crack surfaces while the smeared crack approach, tries to represent the physical discontinuity introduced in a system of cracks by modification of material properties in the zone of cracking [Mirzayee et al., 2010].

From the results review, it's clear that the response of the dam with cracks is higher compared to the results without cracks. The shear stress concentration in the first case, i.e. Koyna model without crack was maximum near to the neck region where the section was changing and damage zone was identified and crack was modelled there. In un-cracked Koyna dam model, the maximum shear stress is 10.33Mpa, Where as in cracked model its 7.71MPa. Principal stresses were also reduced. The horizontal displacement of crest was in the range of 3.6cm. Dynamic analysis results of Koyna dam with crack gave higher value of response about 40 percent increment as of those result values obtained without crack. The analysis result diagrams of cracked Koyna model reveals that at starting stage during earthquake (at 0.005 sec) the crack opens up, then it slowly closes, and at 4.26 sec it again opens, and finally it closes at 9.995sec. i e. The crack behaviors in mixed mode fashion (both opening and closing nature.

4 BEHAVIOUR OF LIFT JOINTS UNDER DYNAMIC EXCITATIONS IN KOYNA DAM

The un-cracked Koyna is modelled here in this section with lift joint. A lift joint is provided at 66.50m from the base of the dam model. The combined effect of water pressure and seismic acceleration (same as previous cases) is taken and transient dynamic analysis is carried out.

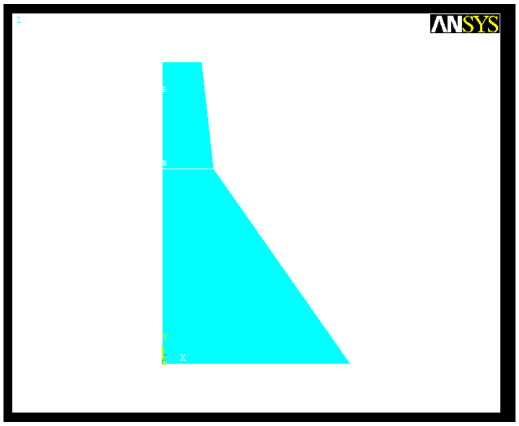

Fig. 9 (A) Koyna Dam model with lift joint

The responses (crest displacements) of Koyna with lift joints compared to Koyna model without crack are lesser, but the dam behaves non-linearly after particular time period (0.24 sec).

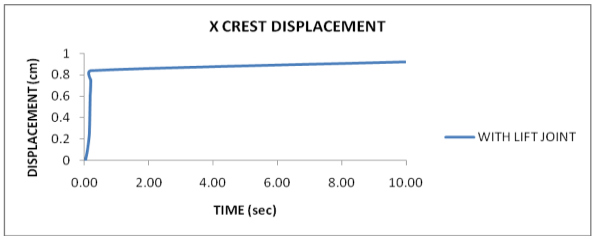

Fig. 9 (B - 1) Koyna Dam model with lift joint – X Crest displacement

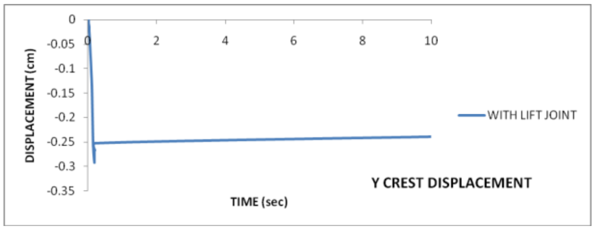

Fig. 9 (B - 2) Koyna Dam model with lift joint – Y Crest displacement

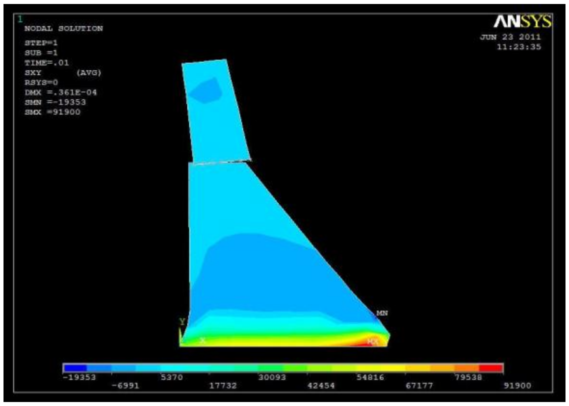

Fig. 9 (C – 1) Shear Stress distribution at 0.01 sec

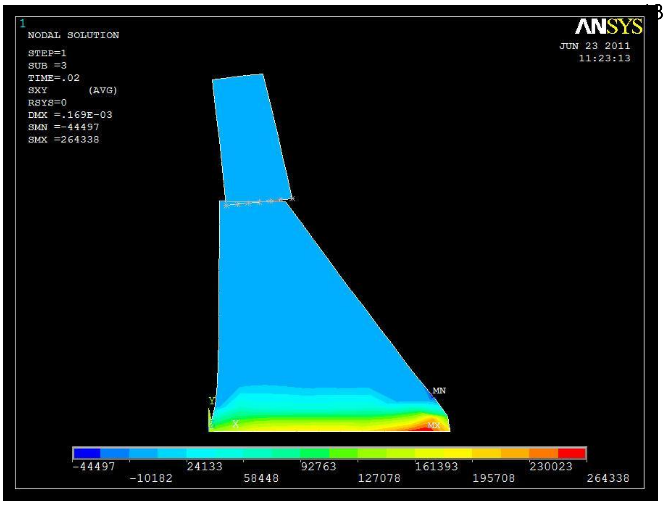

Fig. 9 (C – 2) Shear Stress distribution at 0.02 sec

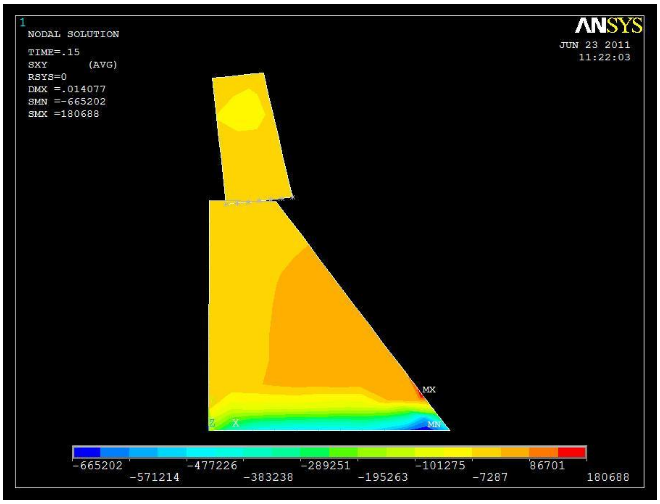

Fig. 9 (C – 3) Shear Stress distribution at 0.15 sec

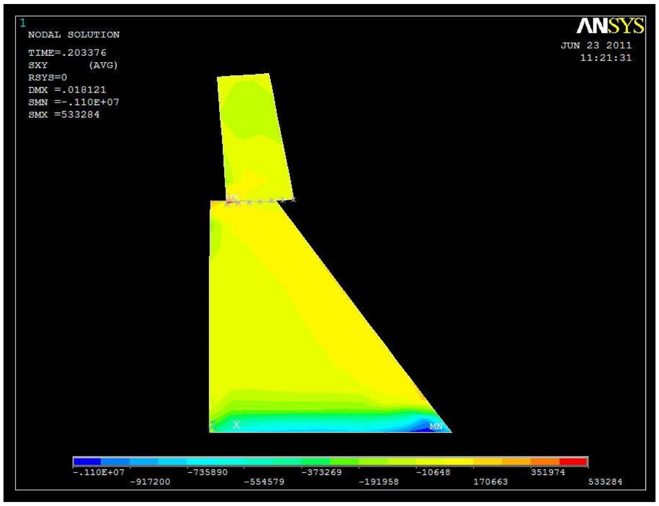

Fig. 9 (C – 4) Shear Stress distribution at 0.20 sec

5 CONCLUSIONS

In this paper the formulation and implementation of embedded finite elements for dynamic crack propagation in concrete were presented. The finite element is implemented using MATLAB for crack propagation in Pine flat dam under dynamic excitations. The results of the analysis were first compared with those obtained from ANSYS to ensure that they are correct.

- An elegant technique for numerical analysis of crack propagation in concrete gravity dams is proposed.
- The cracking behaviour can be captured for aged concrete gravity dams, where both the crack and the bulk are affected by many factors.
- The results obtained from MAT-DAM are consistent with ANSYS.
- Unlike ANSYS and ADINA which lack a proper method of crack propagation in non-symmetric specimens, the present FE technique can track the crack path with complex geometries.
- The present MAT-DAM code is applicable for those cracks which occurs at the first, it shows the initiation and propagation to certain extend but whereas the analytical method (ANSYS) can't predict the path propagation if its mixed mode kind.

From parametric studies it can be concluded that:

- The stresses are reduced for aged concrete whereas the displacement, acceleration and velocity are drastically increased at nodes for same value of damping.
- When the cohesion value is taken as zero the crack pattern is similar to those reported in the literature, this may be due to stress locking in the elements.
- The crack path predict is the same whether aging of concrete considered or not. With an increased length of the crack and bigger damage zone for the case when aging effect is considered.

From the analysis of Koyna it can be concluded:

- Here MAT-DAM code is not used, but from the dynamic analysis results damage zone is identified and crack modeling is done.
- The dynamic analysis results are quite match with literature reference.
- The responses of cracked dam model are higher compared to the dam model without cracks.
- This research shows dynamic analysis results of Koyna dam with crack are increased to 20 - 40 percent of those result values obtained without crack.
- The crack behaviors in mixed mode fashion (both opening and closing nature).
- When dam model is provided with lift joints, the dynamic analysis result shows that it behaves non-linearly after 0.24 sec. So we can't allow the cracks to propagate through the joints since the safety of the structure is not ensured. Provision of shear key will ensure the safety.
- Allowing sliding at the lift joints eliminates stress concentration which would generally occur in the structure. High tensile stresses at the interface resulting from such stress concentration would initiate crack which would propagate along the interface and destroy the proper bond between those faces. So proper safety should be taken at those joints while designing the structure.

FUTURE SCOPE

- The current work extracts the concept of Embedded Finite Element (E-FEM) and that is applied in case of crack propagation in Pine Flat dam using MATLAB and compared with ANSYS. The same work can be extended in Extended Finite Element (X-FEM) also.
- This research work is interested in the first damage zone and concentrating on those cracked (damaged) elements. The work can be extended for second and third damage zone and so on.
- The responses of both Koyna and Pine Flat dam with crack are higher by 10 to 40 percent that of without crack here. By changing the parameters like material (concrete), density, modulus of elasticity, or softening parameter (reinforcement) etc. we can observe the whole response of the structure and how it differs from the present study.
- The present work considers the effect of dam body model alone under combined effect of water pressure and seismic excitation. We can extend this with reservoir effect and foundation interaction.
- The work can be extended for many other dams and can have a comparative study for analysing the behaviour of cracks or fracture those already existing or imposed cracks so that it will be helpful in the field of secure and safety assessment of dams.

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